

Countervailing Vertical Contracting

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29/05/25

Motivation

Ex-ante contracts commonplace in vertical markets.

- A private-cost supplier wants to sell a good to a private-value buyer.
- Before observing cost, must obtain license from a third party to trade.
- Some licensing contracts are complex, involving rebates, royalties, exclusive dealing, and price-maintenance.

Supplier may hold little bargaining power against buyer (e.g., US pharmaceutical distributors & government procurement)

- Our argument: licensor uses **complex contracts to countervail** bargaining power of buyer.
- Price-maintenance clauses, royalties, and performance-based rebates give seller commitment power.

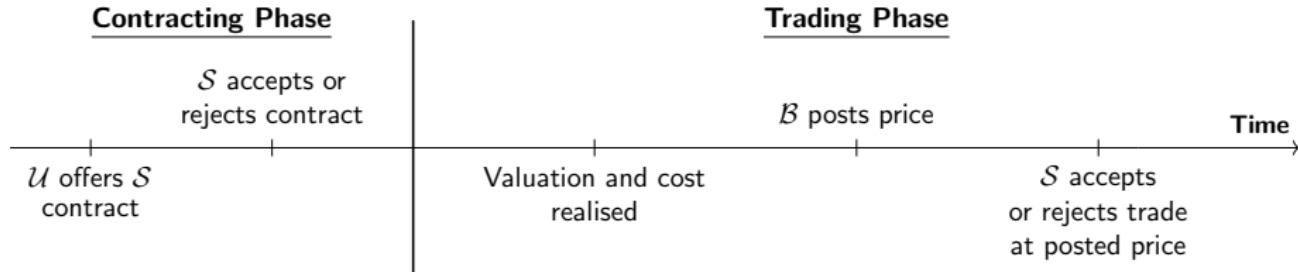
Roadmap

- ① Setup vertical licensing model.
- ② Show WLOG we may restrict attention to two-part royalty contracts involving a fixed fee and royalty.
- ③ Simplify the model: assume either buyer value or supplier cost known.
 - ▶ Highlight 4 observations from reduced model.
- ④ Move to general model with bilateral asymmetric information.
 - ▶ Employ mechanism design to solve for equilibrium.
 - ▶ Show 4 observations generalise to full model.
 - ▶ Gain new insights from generalised model beyond those in reduced model.

Setup I: Overview

Three players: an upstream firm \mathcal{U} , a supplier \mathcal{S} , and a buyer \mathcal{B} . Two phases (1) **Contracting Stage** then (2) **Trading Stage**.

- **Trading Stage:** \mathcal{B} and \mathcal{S} privately know value and cost. \mathcal{B} posts price to \mathcal{S} .
- **Contracting Stage:** \mathcal{U} offers contract to \mathcal{S} which can condition on price posted and if trade occurs. But, not on cost and value. \mathcal{B} observes contract.



Setup II: Costs and Valuations

- Common prior $c \sim G$, $v \sim F$, with $v \perp c$ and F, G smooth. Full-support densities, g and f over $[\underline{c}, \bar{c}]$ and $[\underline{v}, \bar{v}]$, respectively.
- Distributions overlap: $0 < \underline{c} < \underline{v} < \bar{c} < \bar{v}$.

A1: Increasing Hazard Rate of Values

Buyer's value distribution such that $\frac{f(v)}{1-F(v)}$ strictly increasing over $[\underline{v}, \bar{v}]$.

A2: Decreasing Reverse Hazard Rate of Costs

Supplier's cost distribution such that $\frac{g(c)}{G(c)}$ is strictly decreasing over $[\underline{c}, \bar{c}]$.

A3: No Corner Solutions

Distributions close at bottom end: $\underline{c} > \underline{v} - \frac{1}{f(\underline{v})}$.

Setup III: Licensing Contract

Before observing cost and entering downstream market, \mathcal{S} obtains license from \mathcal{U} .

- In **Contracting Stage**, \mathcal{U} offers a contract to \mathcal{S} conditional on price posted ($p \in \mathbb{R}_+$) and whether trade occurs ($x \in \{0, 1\}$),

$$\{0, 1\} \times \mathbb{R}_+ \ni (x, p) \mapsto m(x, p) \in \mathbb{R}$$

Specifies what \mathcal{S} should pay to \mathcal{U} as a function of the outcome of negotiation in the Trading Stage.

- Let \mathbb{M} be all such measurable contracts which admit trading stage equilibria.
- \mathcal{S} decides whether to accept based on rule $A : \mathbb{M} \rightarrow \Delta(\{0, 1\})$.

Observable Contracting

The buyer, \mathcal{B} observes the contract signed, if any, between \mathcal{S} and \mathcal{B} .

Setup IV: Trading

At the start of **Trading Stage**, costs and values realised.

- \mathcal{B} and \mathcal{S} negotiate over sale of single indivisible good.
- \mathcal{B} offers a take-it-or-leave-it price to \mathcal{S} .
- \mathcal{S} either accepts or rejects.

Strategies are,

- \mathcal{B} picks price schedule $p : [\underline{v}, \bar{v}] \times \mathbb{M} \rightarrow \Delta(\mathbb{R}_+)$.
- \mathcal{S} chooses price-acceptance rule $a : [\underline{c}, \bar{c}] \times \mathbb{R}_+ \times \mathbb{M} \rightarrow \Delta(\{0, 1\})$.

Setup V: Payoffs and Equilibrium

- If \mathcal{S} rejects licensing contract, all agents get 0.
- If \mathcal{S} accepts contract, payoffs are,

$$\pi_{\mathcal{B}}(x, p ; v) = x(v - p)$$

$$\pi_{\mathcal{S}}(x, p, m ; c) = x(p - c) - m(x, p)$$

$$\pi_{\mathcal{U}}(x, p, m) = m(x, p)$$

- Define supplier trade surplus as $\hat{\pi}_{\mathcal{S}}(x, p ; c) := x(p - c)$.

Agents cannot signal what they don't know. A **PBE** is,

- A licensing contract $m \in \mathbb{M}$.
- A price-acceptance strategy $a : [\underline{c}, \bar{c}] \times \mathbb{R}_+ \times \mathbb{M} \rightarrow \Delta(\{0, 1\})$ and license contract acceptance strategy $A : \mathbb{M} \rightarrow \Delta(\{0, 1\})$ for \mathcal{S} .
- A price schedule $p : [\underline{v}, \bar{v}] \times \mathbb{M} \rightarrow \Delta(\mathbb{R}_+)$ for \mathcal{B} .

Simplifying the Contract Space

Lemma 1

Without loss of generality, we can restrict attention to **two-part contracts**,

$$m(x, p) = xk(p) + F$$

- F is a **Fixed Fee** paid irrespective of downstream trade outcomes.
- $k : \mathbb{R}_+ \rightarrow \mathbb{R}$ is a **Royalty Payment** paid if trade occurs and can depend on p .

Proof Sketch: Focus on two-part restriction. Decompose any m as

$$m(x, p) = x(m(1, p) - m(0, p)) + m(0, p) =: xk(p) + F(p)$$

- $F(p)$ paid irrespective of whether trade occurs or not \implies sunk cost so price-acceptance decision of $\mathcal{S} \perp F(p)$.
- Risk-neutral agents, can flatten $F(p)$ to $F = \mathbb{E}[F(p)]$.

Implications of Two-Part Contracting

Only royalties, k , are strategically relevant at Trading Stage. F sunk.

- Revenue-maximising \mathcal{U} picks F to hold \mathcal{S} down to its reservation utility.

$$F = \mathbb{E}[\pi_S] = \mathbb{E}[a(p - k(p) - c)]$$

- Therefore, revenue of \mathcal{U} is,

$$\begin{aligned}\mathbb{E}[\pi_U] &= F + \mathbb{E}[ak(p)] \\ &= \mathbb{E}[a(p - c)] \\ &= \mathbb{E}[\hat{\pi}_S]\end{aligned}$$

- \mathcal{U} picks royalties to maximise expected supplier trade surplus.
- Hereafter, suppress reference to F .

One-Sided Imperfect Information

Begin with only one privately informed party, \mathcal{B} or \mathcal{S} .

Definition: p^* -forcing Royalty Scheme

A royalty scheme k is p^* -forcing if it has format

$$k(p) = \begin{cases} \bar{v} & \text{if } p \neq p^* \\ 0 & \text{if } p = p^* \end{cases}$$

- As if \mathcal{U} signs a retail price-maintenance contract requiring \mathcal{S} only trade at p^* .
- \mathcal{S} obligated to pay **liquidated damages** for contractual breach.
- Gives \mathcal{S} credible commitment power to only accept trade at p^* .
- \mathcal{B} either posts price p^* or cannot trade.

One-Sided Imperfect Information: Optimal Contract

If imperfect information is one-sided, the optimal contract is forcing. Define

$$\psi_{\mathcal{B}}(v) = v - \frac{1 - F(v)}{f(v)}$$

Proposition 1

Known Value: If \mathcal{B} 's valuation distribution is degenerate on $v \in [\underline{v}, \bar{v}]$, then \mathcal{U} offers a v -forcing contract. Trade if and only if $v \geq c$

Known Cost: If \mathcal{S} 's cost distribution is degenerate on $c \in [\underline{c}, \bar{c}]$, then \mathcal{U} offers a $\psi_{\mathcal{B}}^{-1}(c)$ -forcing contract. Trade if and only if $v \geq \psi_{\mathcal{B}}^{-1}(c)$.

All supplier types agree on the price they prefer. Royalty scheme forces this price.

One-Sided Imperfect Information: Observations

- \mathcal{U} makes monopoly profit with forcing contracts.
- p^* -forcing contracts require no on-path royalty payments.

Observation 1

\mathcal{U} makes monopoly profit \implies buyer power **fully countervailed by royalties**.

Observation 2

Expected royalty payments are 0.

One-Sided Imperfect Information: Observations

Compare welfare with royalties to when $k \equiv 0$.

- If buyer valuation known, royalties necessarily **improve welfare**.
 - ▶ With royalties, trade is efficient.
- If supplier cost known, royalties necessarily **harm welfare**.
 - ▶ Without royalties, trade is efficient.
- Buyer could have posted supplier-optimal prices but didn't.
 - ▶ $\Rightarrow \mathcal{B}$ cannot benefit from the royalty contract

Observation 3

The welfare effect of royalties is ambiguous.

Observation 4

The buyer is always worse off from the introduction of royalties.

Bilateral Imperfect Information

Now turn to general problem, $c \sim G$, $v \sim F$, over non-null intervals $[\underline{c}, \bar{c}]$, and $[\underline{v}, \bar{v}]$, respectively.

- Forcing contracts no longer optimal. Different cost types disagree about optimal price.
- Instead, \mathcal{U} permits some pricing discretion to \mathcal{B} .

Given royalty scheme k , trading-stage behaviour is

- \mathcal{S} follows *cost-cutoff* acceptance rule, accepts iff $p - k(p) \geq c$.
- \mathcal{B} picks price $p_k(v) \in \arg \max_p G(p - k(p))(v - p)$

For \mathcal{U} , full shape of k matters. High dimensionality resists direct search.

Bounding Trade Surplus

Use mechanism design to place upper bound on equilibrium payoffs and show achievable by some royalty contract.

- Identify any trading-stage equilibrium with a direct incentive compatible mechanism $\langle q, t \rangle$ where
 - ▶ $q(c, v)$ is probability of trade.
 - ▶ $t(c, v)$ is transfer from \mathcal{B} to \mathcal{S} .
 - ▶ Define interim counterparts for \mathcal{B} and \mathcal{S} as $(Q_{\mathcal{B}}(v), T_{\mathcal{B}}(v))$ and $(Q_{\mathcal{S}}(c), T_{\mathcal{S}}(c))$, respectively.

Explicitly, given equilibrium price schedule $p_k(v)$

- $q(c, v) = 1$ iff $p_k(v) - k(p_k(v)) \geq c$
- $t(c, v) = q(c, v)p_k(v)$

Bounding Trade Surplus

From Myerson (1981), $(Q_B(v), T_B(v))$ incentive compatible for \mathcal{B} iff

$$Q_B(v) \text{ increasing} \quad (1)$$

$$T_B(v) = Q_B(v)v - \int_{\underline{v}}^v Q_B(x)dx - \Pi_B(\underline{v}) \quad (2)$$

$$\Pi_B(\underline{v}) \geq 0 \quad (3)$$

Using (2), revenue of \mathcal{U} is affine in Q_B, Q_S ,

$$\begin{aligned} R(\langle q, t \rangle) &:= \int_{\underline{c}}^{\bar{c}} \int_{\underline{v}}^{\bar{v}} t(c, v) - q(c, v) cdF dG \\ &= -\Pi_B(\underline{v}) + \int_{\underline{v}}^{\bar{v}} Q_B(v) \psi_B(v) dF - \int_{\underline{c}}^{\bar{c}} Q_S(c) cdG \end{aligned}$$

Bounding Trade Surplus

$$R(\langle q, t \rangle) = -\Pi_B(\underline{v}) + \int_{\underline{v}}^{\bar{v}} Q_B(v) \psi_B(v) dF - \int_{\underline{c}}^{\bar{c}} Q_S(c) c dG$$

As $q(c, v)$ induced by equilibrium, $q(c, v) = \mathbb{1}\{h(v) \geq c\}$ for some increasing function h . Allows us to remove Q_S from $R(\langle q, t \rangle)$.

Lemma 3: A Revenue-Equivalence Result

In any equilibrium mechanism $\langle q, t \rangle$, the revenue of \mathcal{U} is

$$-\Pi_B(\underline{v}) + \int_{\underline{v}}^{\bar{v}} [Q_B(v) \psi_B(v) - C(Q_B(v))] dF$$

where $C(Q) = Q\mathbb{E}[c \mid c \leq G^{-1}(Q)]$ is a differentiable, increasing, and convex function defined in $[0, \mathbb{E}[c]]$, with first derivative $C'(Q) = G^{-1}(Q)$.

Convex cost, C , as trade with the lowest cost types first.

Bounding Trade Surplus

$$\max_{Q_B} -\Pi_B(\underline{v}) + \int_{\underline{v}}^{\bar{v}} [Q_B(v)\psi_B(v) - C(Q_B(v))] dF$$

st. $Q_B(v)$ increasing & $\Pi_B(\underline{v}) \geq 0$

Analogous to Mussa and Rosen (1978) 2nd-degree price discrimination.
Ignoring monotonicity, pointwise maximisation gives $\Pi_B(\underline{v}) = 0$ and

$$C'(Q_B^*(v)) = \psi_B(v) \iff Q_B^*(v) = G(\psi_B(v))$$

Revenue is bounded above by $\int_{\underline{v}}^{\bar{v}} Q_B^*(v)\psi_B(v) - C(Q_B^*(v))dF$.

- For (k^*, p_k^*) to achieve bound, require

$$G(Q_B^*(v))p_k^*(v) = Q_B^*(v)v - \int_{\underline{v}}^v Q_B^*(x)dx$$

$$G(p_k^*(v) - k^*(p_k^*(v))) = Q_B^*(v)$$

The Optimal Contract

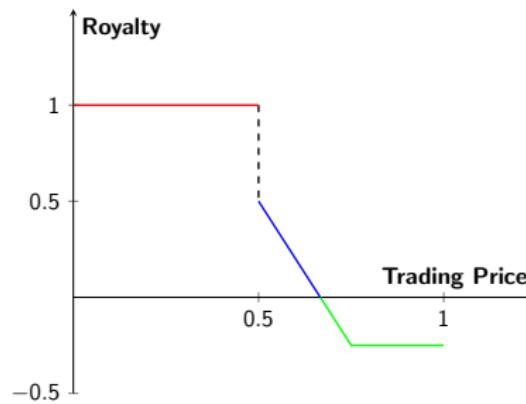


Figure 1: $c \sim U[0, 1]$, $v \sim U[0, 1]$.

- Price maintenance: No accepting prices less than $1/2$.
- Negative royalties: Subsidise accepting prices greater than $2/3$.
- Decreasing: High royalties at low prices, low/negative at high prices.

The Optimal Contract

Theorem 1

① The contract

$$k^*(p) = \begin{cases} \bar{v} & p < \psi_{\mathcal{B}}^{-1}(\underline{c}) \\ p - \psi_{\mathcal{B}}((p_k^*)^{-1}(p)) & p \geq \psi_{\mathcal{B}}^{-1}(\underline{c}) \end{cases}$$

$$F^* = \int_{\underline{c}}^{\bar{c}} \int_{\underline{v}}^{\bar{v}} 1_{\{\psi_{\mathcal{B}}(v) \geq c\}} (\psi_{\mathcal{B}}(v) - c) dF$$

is proposed by \mathcal{U} and accepted by \mathcal{S} .

② The pricing decision of \mathcal{B} is

$$p_{k^*}^*(v) = \mathbb{E}[\psi_{\mathcal{B}}^{-1}(c) \mid c \leq \psi_{\mathcal{B}}(v)]$$

③ The acceptance decision of \mathcal{S} is given by

$$a_{k^*}(c, p) = \mathbb{1}_{\{c \leq p - k^*(p)\}}.$$

The Optimal Contract: Observation 1

To what extent does the optimal contract 'countervail' the power of \mathcal{B} ?

Two notions of \mathcal{S} having power:

- ① \mathcal{S} can commit ex-ante to a price-acceptance strategy, gives $\pi(\text{Commit})$ revenue.
- ② \mathcal{S} picks the mechanism in the downstream market, gives $\pi(\text{Mech})$.
 - ▶ \mathcal{S} would post TIOLI prices $p_{\mathcal{S}}(c) = \psi_{\mathcal{B}}^{-1}(c)$.

$$\pi(\text{Mech}) = \int_{\underline{c}}^{\bar{c}} \int_{\underline{v}}^{\bar{v}} 1_{\{\psi_{\mathcal{B}}(v) \geq c\}} (\psi_{\mathcal{B}}^{-1}(c) - c) dF dG$$

Optimal royalties earn $\pi(k^*)$. Naturally $\pi(k^*) \leq \pi(\text{Commit}) \leq \pi(\text{Mech})$.

Observation 1

$\pi(k^*) = \pi(\text{Commit}) = \pi(\text{Mech})$. That is, optimal royalties **completely countervail** the market power of \mathcal{B} .

The Optimal Contract: Observation 1

- Can show algebraically that $\pi(k^*) = \pi(\text{Mech})$, i.e.,

$$\underbrace{\int_{\underline{c}}^{\bar{c}} \int_{\underline{v}}^{\bar{v}} 1_{\{\psi_B(v) \geq c\}} (\psi_B(v) - c) dF dG}_{\text{Royalties}} = \underbrace{\int_{\underline{c}}^{\bar{c}} \int_{\underline{v}}^{\bar{v}} 1_{\{\psi_B(v) \geq c\}} (\psi_B^{-1}(c) - c) dF dG}_{\text{TIOLI price offers}}$$

- ▶ Start with LHS. Apply change of variable $t = \psi_B^{-1}(c)$, then simplify.
- Or, notice that in constructing a bound on trade surplus, only ever used that mechanism was from an equilibrium to conclude cost-cutoff rule $q(c.v) = 1\{h(v) \geq c\}$.
 - But, any extreme point in the space of mechanisms has this property (Yang and Yang 2025).
 - So, bound is over all mechanisms, not just those induced by equilibria.

The Optimal Contract: Observation 2

Observation 2

Expected royalty payments are 0.

Under optimal royalties $p_k^*(v) - k^*(p_k^*(v)) = \psi_B(v)$, so fixed fee is

$$\begin{aligned} F^* &= \int_{\underline{c}}^{\bar{c}} \int_{\underline{v}}^{\bar{v}} 1_{\{\psi_B(v) \geq c\}} [p_k^*(v) - k^*(p_k^*(v)) - c] dFdG \\ &= \int_{\underline{c}}^{\bar{c}} \int_{\underline{v}}^{\bar{v}} 1_{\{\psi_B(v) \geq c\}} [\psi_B(v) - c] dFdG \end{aligned}$$

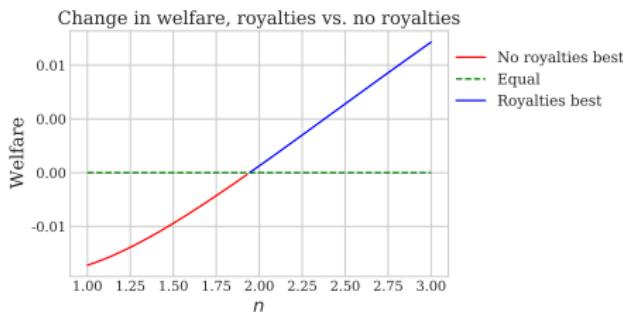
Revenue of \mathcal{U} achieves trade surplus bound,

$$\begin{aligned} R &= F^* + \int_{\underline{c}}^{\bar{c}} \int_{\underline{v}}^{\bar{v}} 1_{\{\psi_B(v) \geq c\}} k^*(p_k^*(v)) dFdG = \int_{\underline{c}}^{\bar{c}} \int_{\underline{v}}^{\bar{v}} 1_{\{\psi_B(v) \geq c\}} [\psi_B(v) - c] dFdG \\ &\implies \int_{\underline{c}}^{\bar{c}} \int_{\underline{v}}^{\bar{v}} 1_{\{\psi_B(v) \geq c\}} k^*(p_k^*(v)) dFdG = 0 \end{aligned}$$

The Optimal Contract: Observation 3

Observation 3

The welfare effect of royalties is ambiguous.



Costs on $[0, 1]$ with $G(x) = \frac{1-e^{-2x}}{1-e^{-2}}$.
Valuations on $[0, 1]$ with $F(c; n) = x^n$.

The Optimal Contract: Observation 4

Observation 4

Royalties always harm the buyer.

With royalties, welfare and revenue of \mathcal{U} is

$$W = \int_{\underline{c}}^{\bar{c}} \int_{\underline{v}}^{\bar{v}} 1_{\{\psi_B(v) \geq c\}} (v - c) dFdG$$

$$R(\langle q, t \rangle) = \int_{\underline{c}}^{\bar{c}} \int_{\underline{v}}^{\bar{v}} 1_{\{\psi_B(v) \geq c\}} (\psi_B(v) - c) dFdG$$

As \mathcal{S} is held down to 0 utility, with royalties \mathcal{B} obtains $BS = W - R$,

$$\underbrace{\int_{\underline{c}}^{\bar{c}} \int_{\underline{v}}^{\bar{v}} 1_{\{\psi_B(v) \geq c\}} (v - \psi_B(v)) dFdG}_{\text{With Royalties}} \leq \underbrace{\max_{p: [\underline{v}, \bar{v}] \rightarrow [0,1]} \int_{\underline{c}}^{\bar{c}} \int_{\underline{v}}^{\bar{v}} 1_{\{p(v) \geq c\}} (v - p(v)) dFdG}_{\text{Without Royalties}}$$

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THANK YOU

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